



### ALGORITHM CORRECTNESS PROOFS

P.1. Find the precondition of the following piece of code:  $y:=2/(2*b+c)$ ;  $x:=1-b*y$  if its postcondition is:  $\{(x+b*y=u) \wedge (c*y-2*x=0)\}$ .

P.2. Let A and B be the initial values of “a” and “b” respectively. Write a piece of code with the following postcondition:  $\{(a=A+B) \wedge (b=A-B)\}$  and prove its correctness.

P.3. Prove that the assertion:  $\{sum=j*(j-1)/2\}$  is the invariant of the following piece of code:  $sum:=sum+j$ ;  $j:=j+1$ .

P.4. Prove the partial correctness of the following piece of code:

```
Sum:=0; j:=1;
While (j<>c) do
  Begin
    Sum:=sum+j; j:=j+1
  End;
{sum=c*(c-1)/2}
```

P.5. Prove the partial correctness of the following piece of code:

```
{n≥0}
i:=1;
while i≤n do
  begin
    a[i]:=b[i];
    i:=i+1
  end;
{ $\bigwedge_{i=1}^n (a[i]=b[i])$ }
```

P.6. Let A and B be the initial values of “a” and “b” respectively. Prove the partial correctness of the following code that works out the addition of two integer numbers:

```
while a≠0 do
  begin
    a:=a-1;
    b:=b+1
  end;
{(b=A+B)  $\wedge$  (a=0)}
```

P.7. Prove the partial correctness of the following code that works out the division of two integer numbers (A and B):

```

q:=0; r:=A;
while r≥B do
  begin
    r:=r-B;
    q:=q+1
  end;
{(q*B+r=A) ∧ (0≤r<B)}

```

P.8. Let A and B be the initial values of “a” and “b” respectively. Prove the partial correctness of the following code that works out the Maximum Common Divisor (MCD) of two integer numbers:

```

while a≠b do
  if a>b then
    a:=a-b
  else
    b:=b-a

```

For proving the partial correctness, take into account the following features of the MCD:

- if  $a > b$  then  $\text{MCD}(a,b) = \text{MCD}(a-b,b)$
- $\text{MCD}(a,b) = \text{MCD}(b,a)$
- $\text{MCD}(a,a) = a$

P.9. Prove the partial correctness of the following code that works out  $A^N$ , where A and N are integer numbers:

```

q:=1; z:=A; w:=N;
while w>0 do
  begin
    w:=w-1;
    q:=q*z
  end;
{(q=A^N) ∧ (w=0)}

```

P.10. Prove the partial correctness of the following code that works out  $\sum_{i=1}^n i!$ .

```

i:=1; sum:=0; f:=1;
while i<=n do
  begin
    sum:=sum+f;
    i:=i+1;
    f:=f*i
  end;

```

end;